

ARITH 2024



# Combining Power and Arithmetic Optimization via Datapath Rewriting

Samuel Coward, Emiliano Morini, Theo Drane, George A. Constantinides

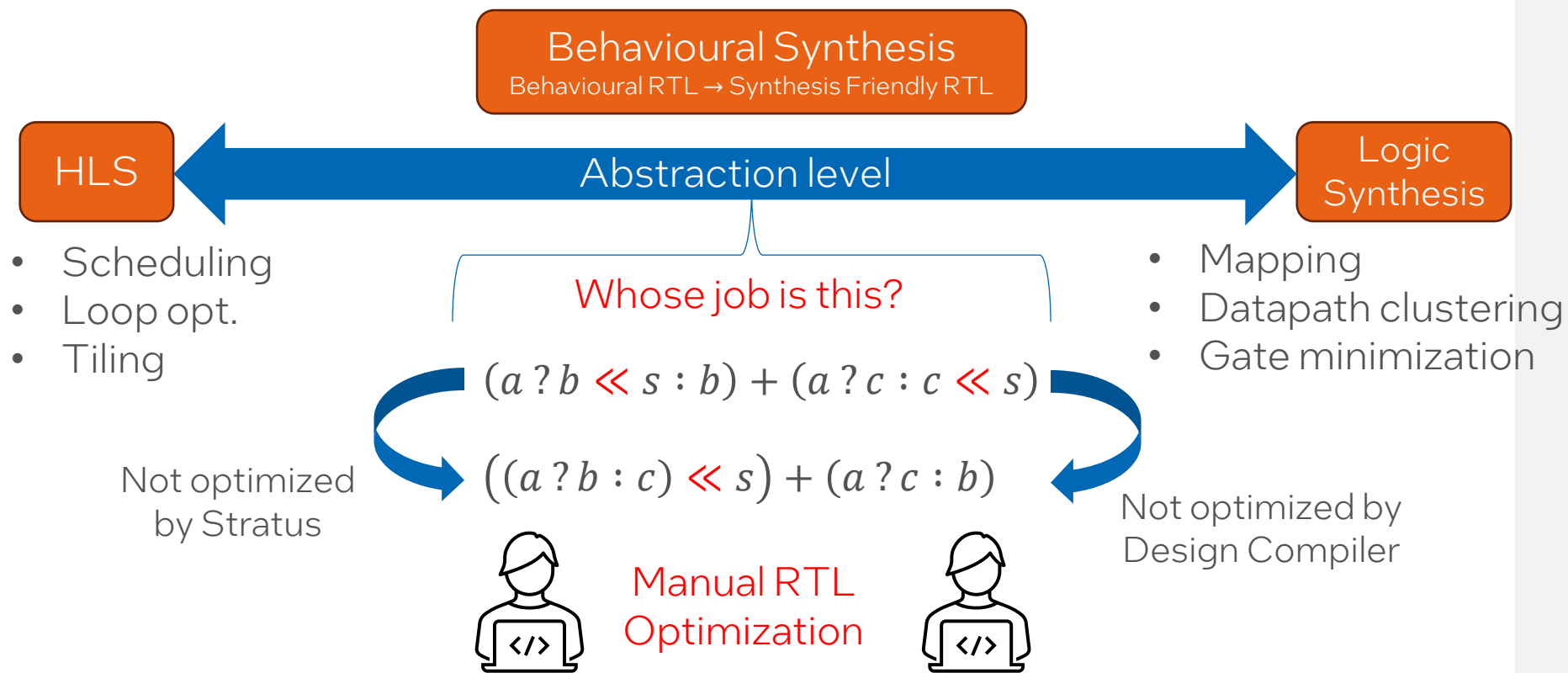
Numerical Hardware Group, Intel Corporation  
EEE, Imperial College London



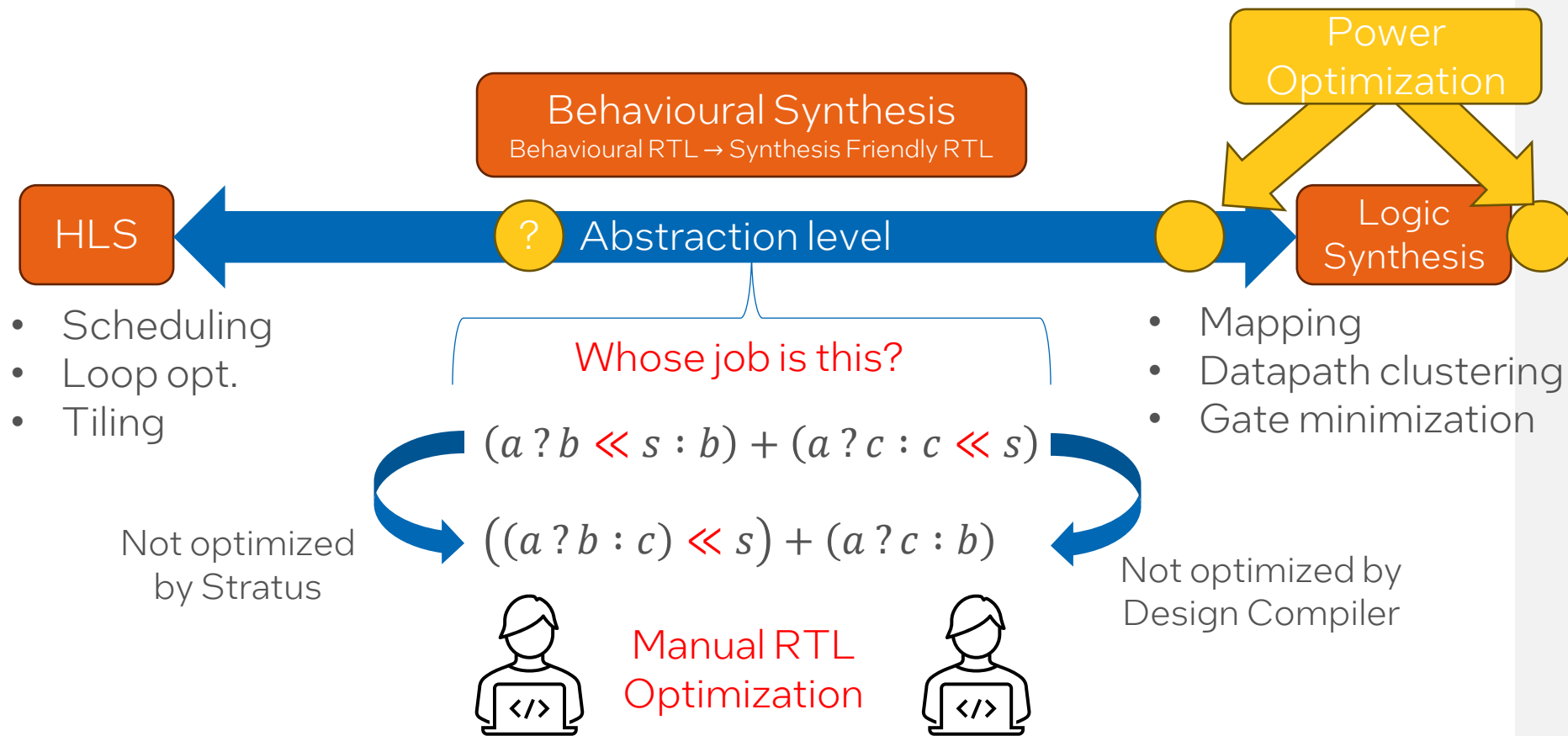
# Motivation



# Motivation



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# E-Graphs

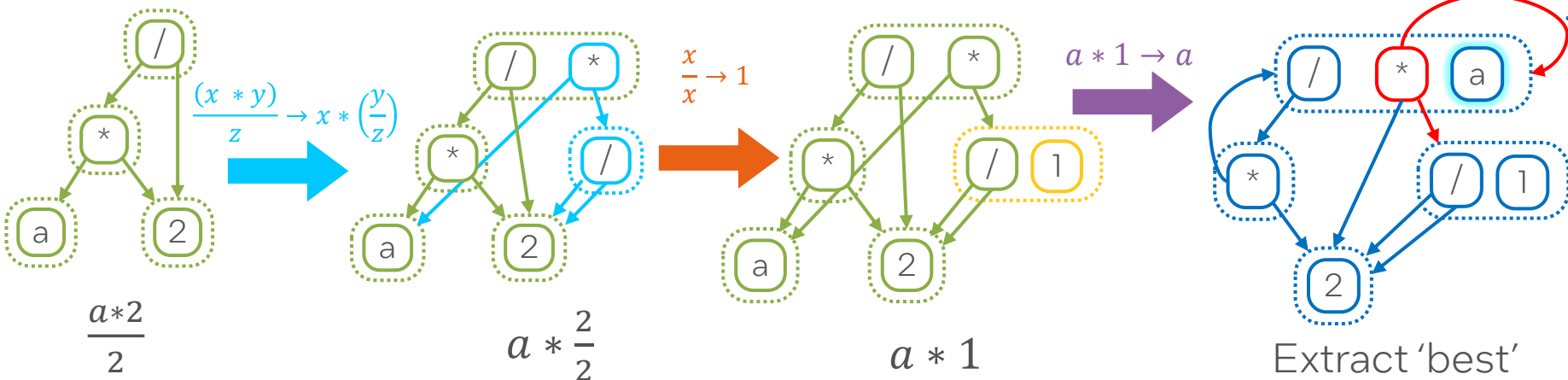
☺ egg: e-graphs good

- Compact representation of equivalent designs
- Maintains history
- Combine program analysis & rewriting
- Constructive rewrite application – phase ordering

Z3

CVC5

fastly



## E-Syn: E-Graph Rewriting with Technology-Aware Cost Functions for Logic Synthesis

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HKUST(GZ)  
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## Equality Saturation for Datapath Synthesis: A Pathway to Pareto Optimality

Ecenur Ustun<sup>1</sup>, Cunxi Yu<sup>2</sup>, and Zhiru Zhang<sup>1</sup>

## ESFO: Equality Saturation for FIRRTL Optimization

Yan Pi

Hongji Zou

Tun Li

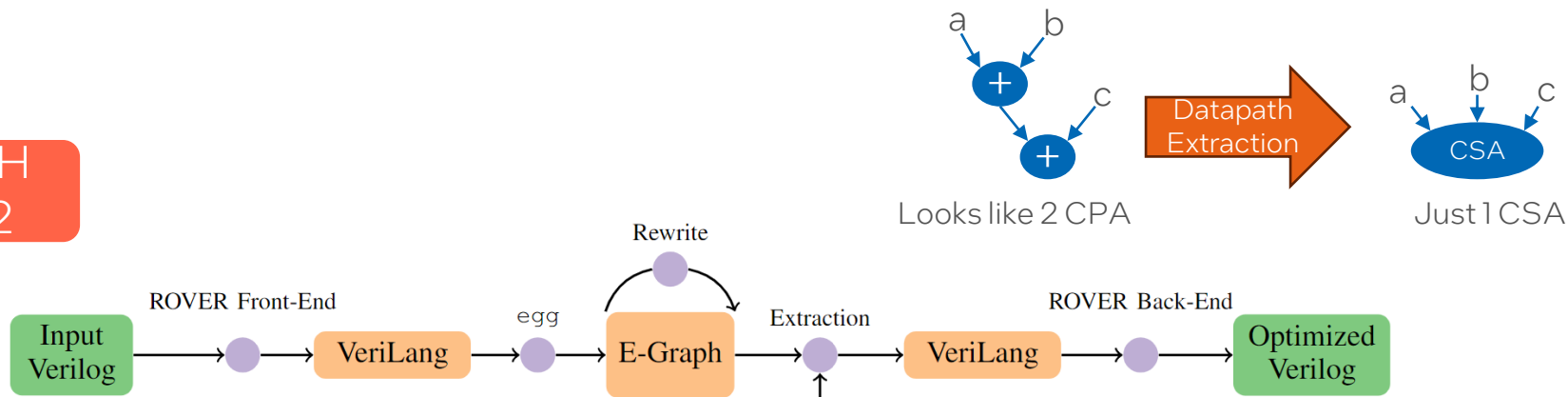
## There and Back Again: A Netlist's Tale with Much Egraphin'

Gus Henry Smith<sup>†</sup>, Zachary D. Sisco<sup>‡</sup>, Thanawat Techaumnaiwit<sup>‡</sup>, Jingtao Xia<sup>‡</sup>, Vishal Canumalla<sup>†</sup>, Andrew Cheung<sup>†</sup>, Zachary Tatlock<sup>†</sup>, Chandrakana Nandi<sup>§</sup>, Jonathan Balkind<sup>‡</sup>

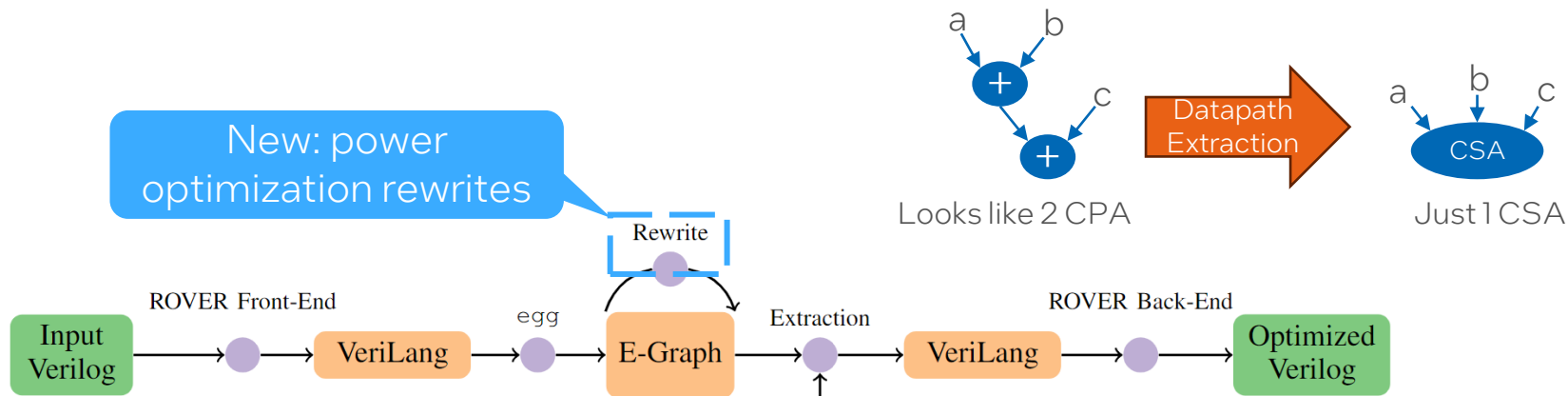
<sup>†</sup> University of Washington <sup>‡</sup> University of California, Santa Barbara <sup>§</sup> Certora, Inc.

# ROVER – RTL Opt. via Verified E-Graph Rewriting

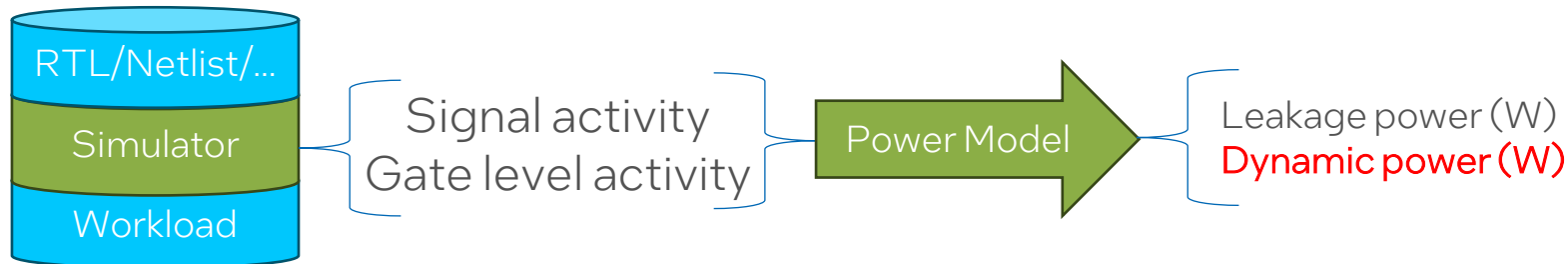
ARITH  
2022



# ROVER – RTL Opt. via Verified E-Graph Rewriting

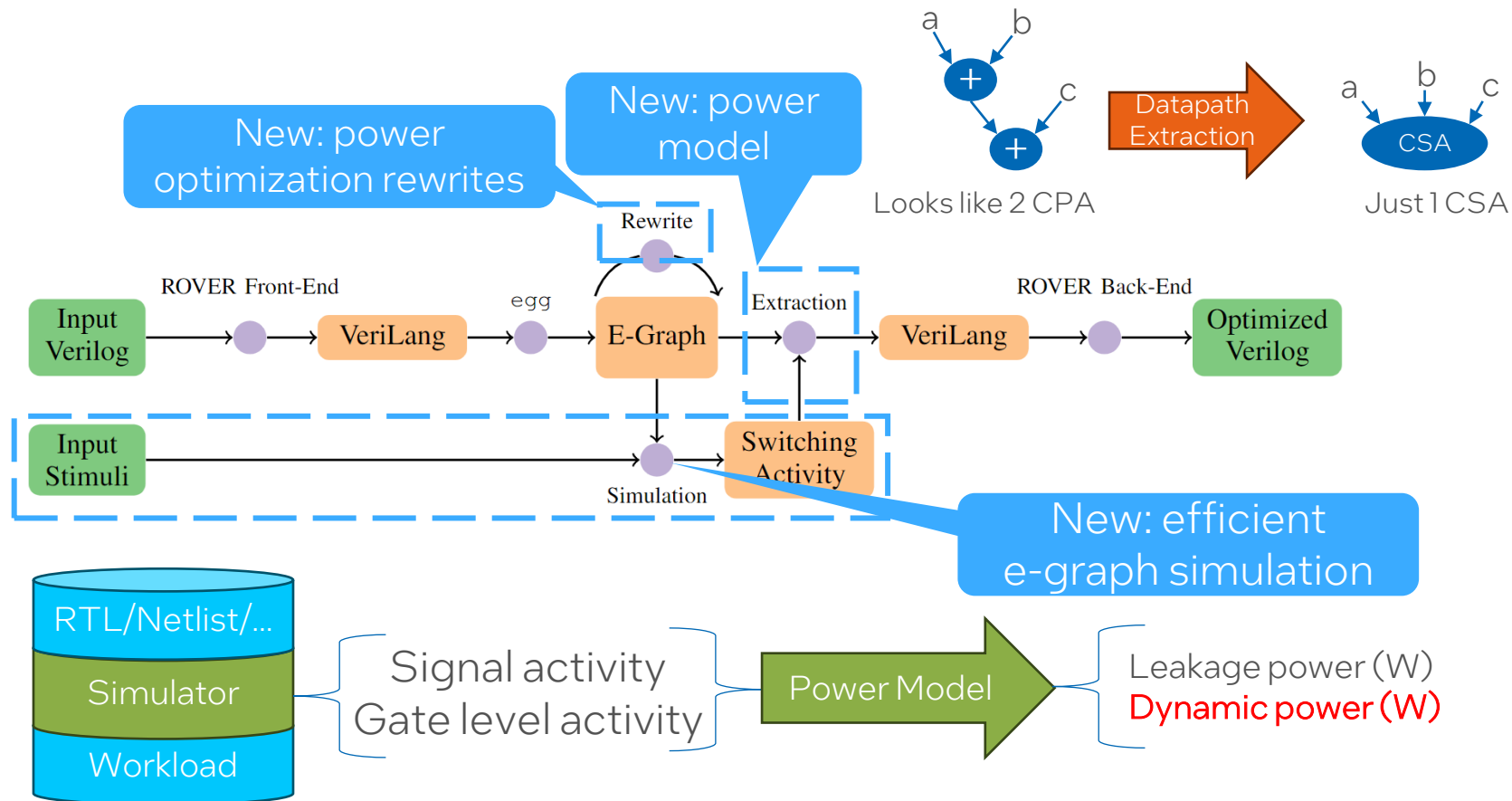


## Measuring Power





# ROVER – RTL Opt. via Verified E-Graph Rewriting



# Don't Cares – hardware is not software

```
if a:  
    f(x)  
else:  
    g(y)
```

**Software** – lazy evaluation:  
f(x) only computed if a=true

**Hardware** – eager evaluation:  
f(x) and g(y) always evaluated

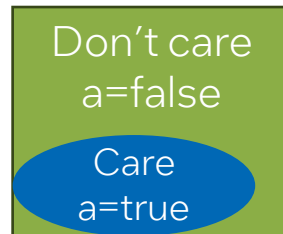
- Muxes introduce potential redundancy
- Redundant computations = wasted power

DAC 2023 – exploit don't care for performance

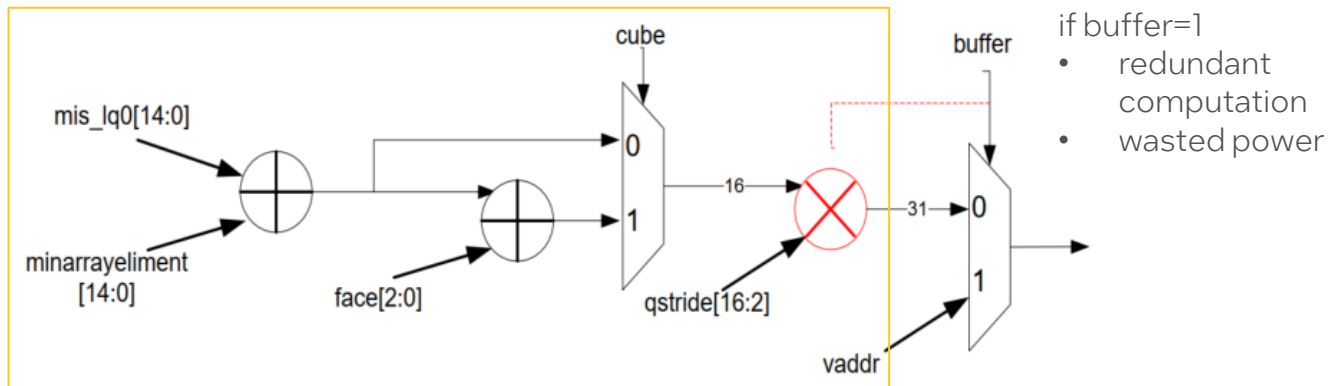
$$a ? f(x) : g(y) \rightarrow a ? f'(x) : g(y)$$

Such that:

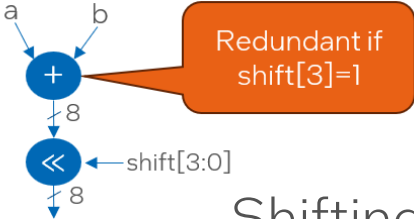
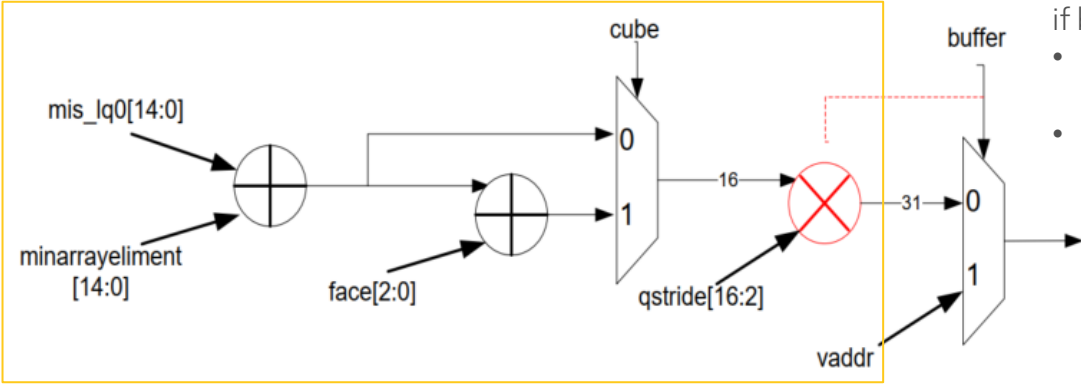
$$f'(x) = \begin{cases} f(x) & \text{if } a \\ \text{don't care} & \text{else} \end{cases}$$



# Redundant Computation & Wasted Watts

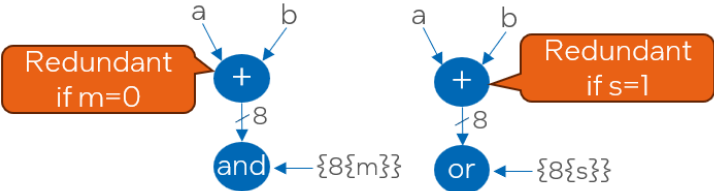
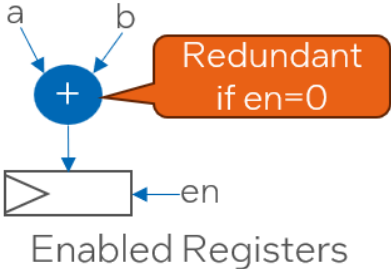
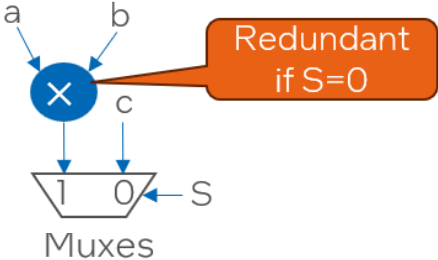


# Redundant Computation & Wasted Watts



Shifting

## Sources of Redundancy

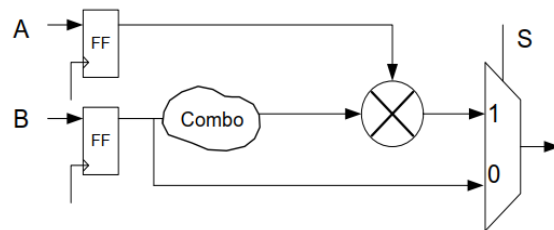


Masking/Saturating

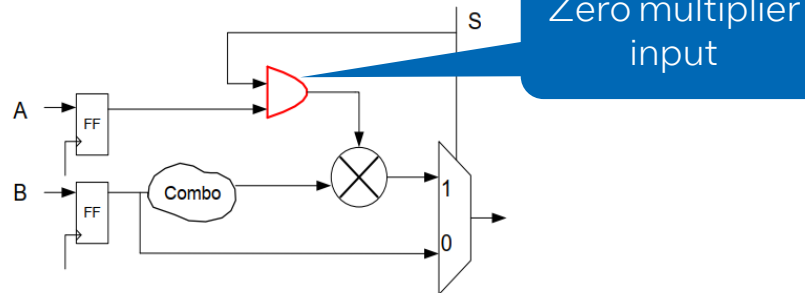
# 1) Data Gating/Operand Isolation – Zeroing Datapath

- Create a mask from select signal
- Bitwise AND with datapath inputs

## Original



## Low Power

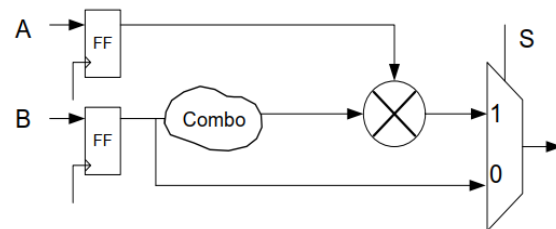


# 1) Data Gating/Operand Isolation – Zeroing Datapath

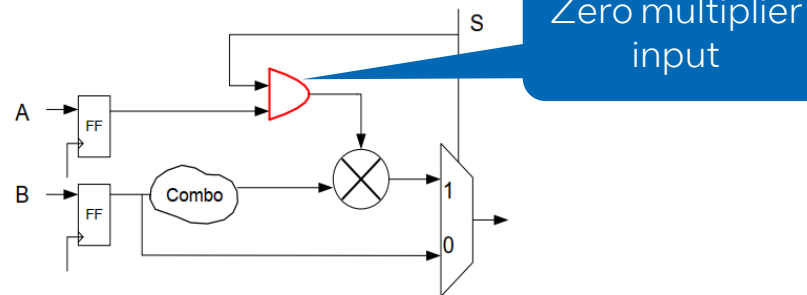
- Create a mask from select signal
- Bitwise AND with datapath inputs

Left	Right
$s ? b : c$	$s ? (b \& \{w_b\{s\}\}) : (c \& \{w_c\{\bar{s}\}\})$
$(a \text{ op } b) \& \{w_o\{s\}\}$	$(a \& \{w_o\{s\}\}) \text{ op } (b \& \{w_b\{s\}\})$
$(a \& \{w_a\{s_1\}\}) \& \{w_a\{s_2\}\}$	$(a \& \{w_a\{s_1 \& s_2\}\})$

## Original



## Low Power



# 1) Data Gating/Operand Isolation – Zeroing Datapath

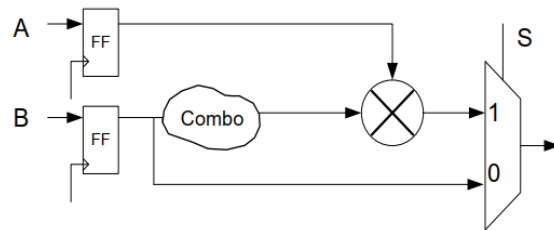
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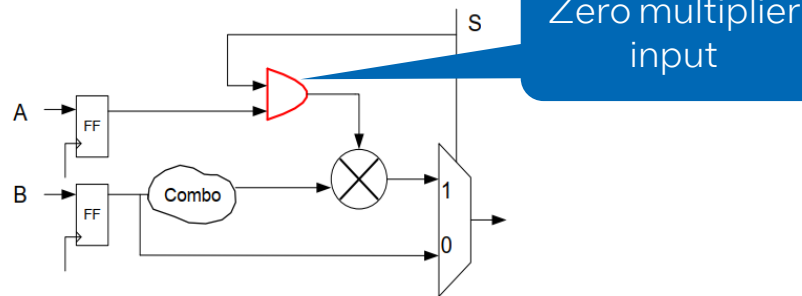
## Considerations:

- Toggle rate of select signal S
- Toggle rate of data signals A,B
- Timing impact & gate count

## Original



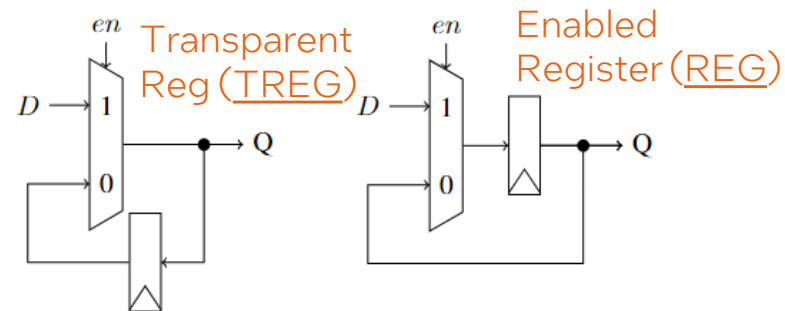
## Low Power



Zero multiplier input

## 2) Transparent Latch/Register – Operand Isolation

- When enabled is transparent
- When disabled input is “frozen” – no power consumed in datapath

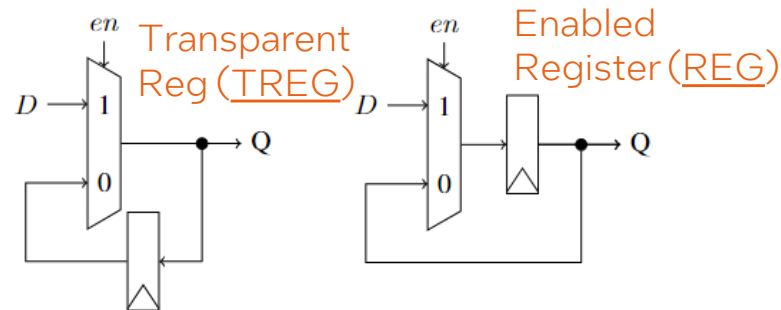
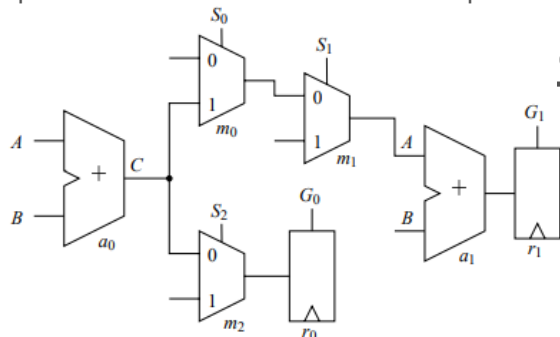




## 2) Transparent Latch/Register – Operand Isolation

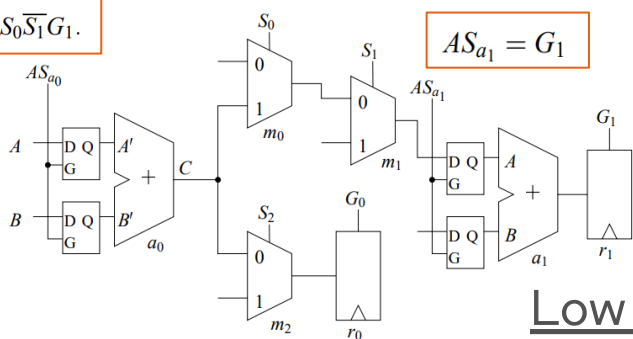
- When enabled is transparent
- When disabled input is “frozen” – no power consumed in datapath

Original



$$\overline{S_2}G_0 + S_0\overline{S_1}G_1$$

$$AS_{a_1} = G_1$$

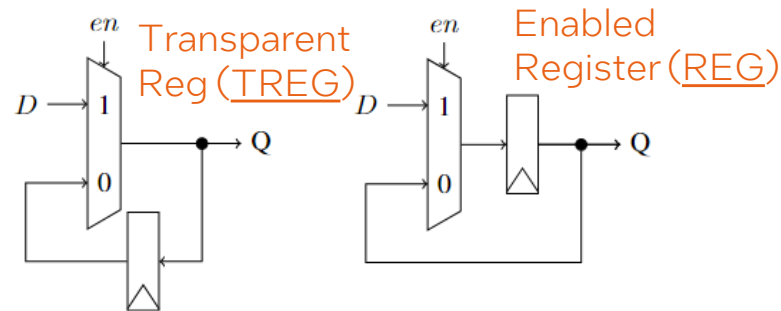
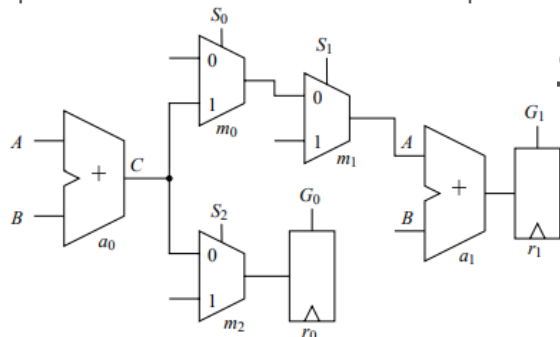


Low Power

## 2) Transparent Latch/Register – Operand Isolation

- When enabled is transparent
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Original

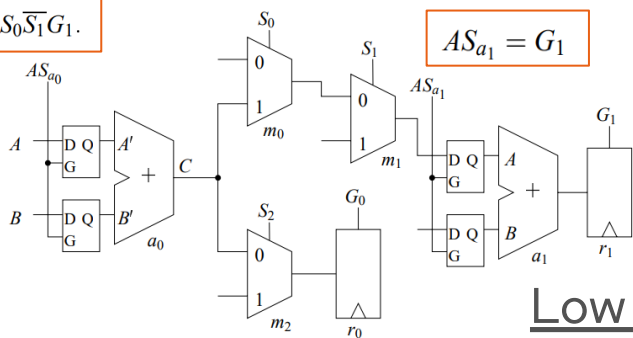


Left	Right
$s ? b : c$	$s ? TREG(b, s) : TREG(c, \bar{s})$
$TREG(a \text{ op } b, s)$	$TREG(a, s) \text{ op } TREG(b, s)$
$REG(a, en)$	$REG(TREG(a, en), en)$

### Considerations:

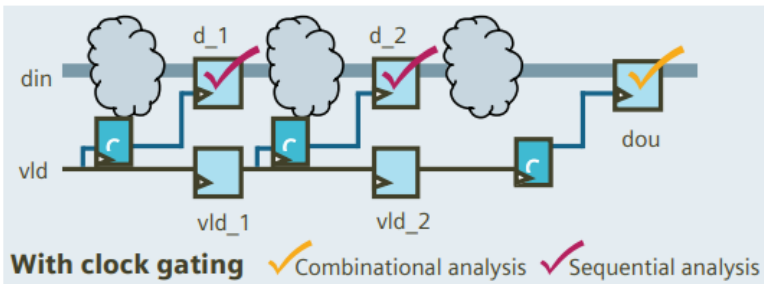
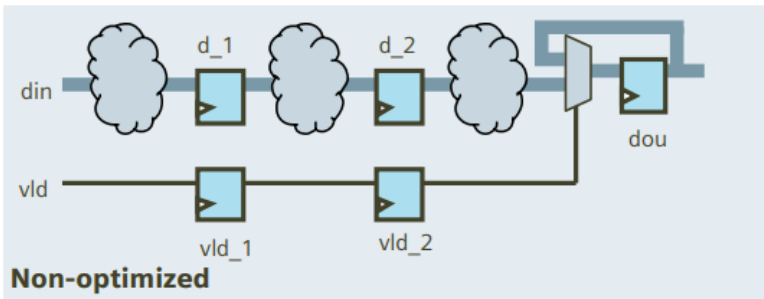
- Clock gating available?
- Timing impact & gate count

Low Power



### 3) Clock Gating – Switching off Datapath

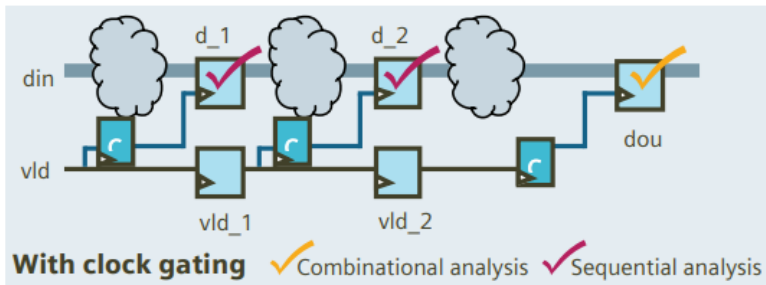
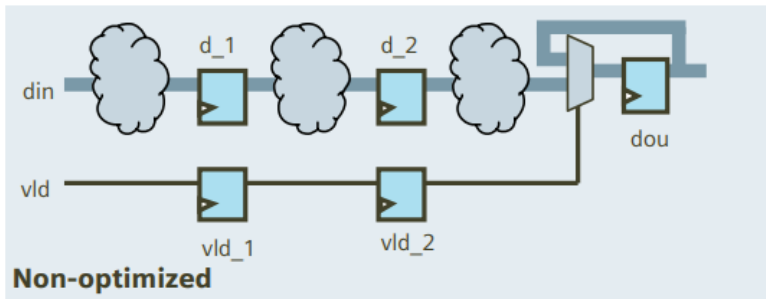
- Modify register enable signals
- Construct *observability don't care* conditions



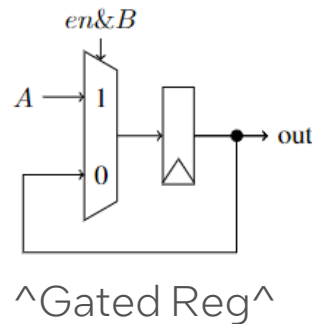
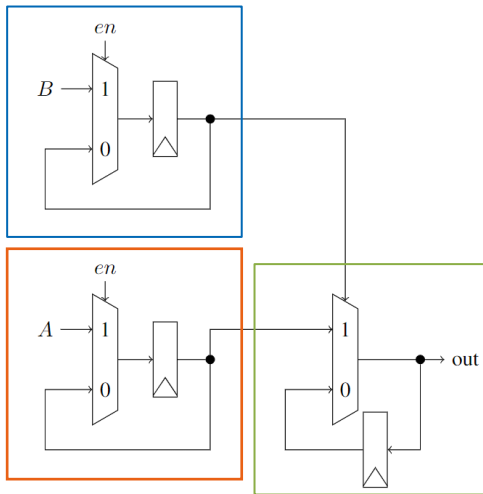
[Siemens White Paper](#)

### 3) Clock Gating – Switching off Datapath

- Modify register enable signals
- Construct *observability don't care* conditions



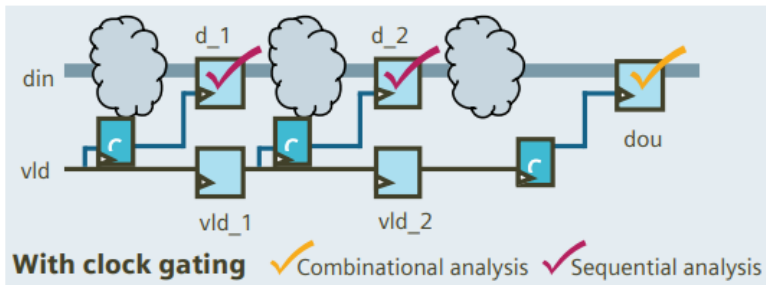
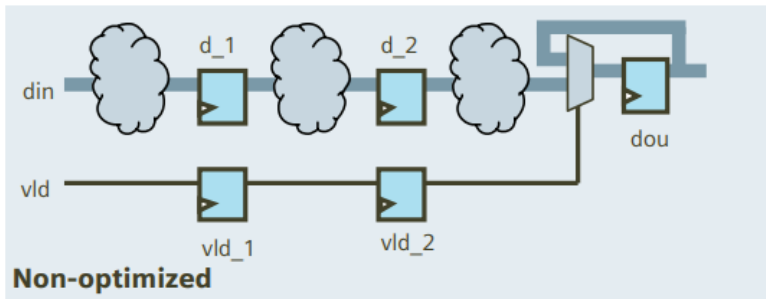
$$TREG(REG(A, en), REG(B, en)) \rightarrow REG(A, en \& B)$$



[Siemens White Paper](#)

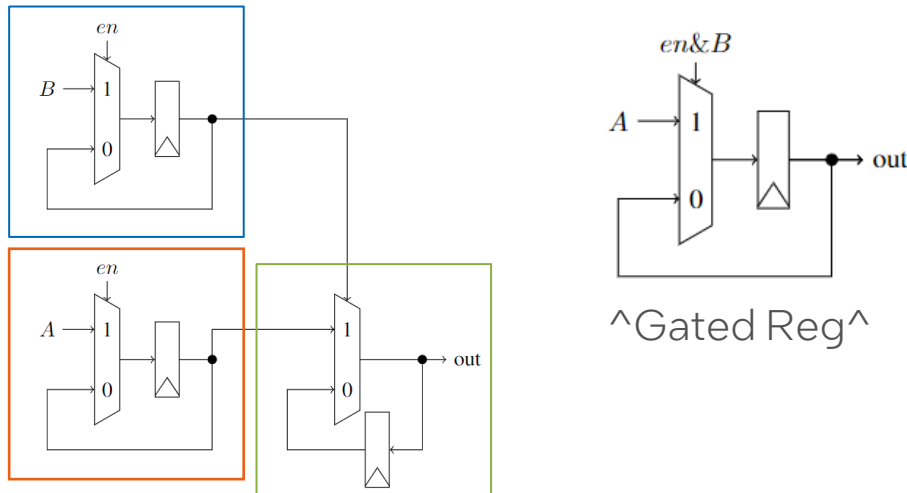
### 3) Clock Gating – Switching off Datapath

- Modify register enable signals
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[Siemens White Paper](#)

$$TREG(REG(A, en), REG(B, en)) \rightarrow REG(A, en \& B)$$

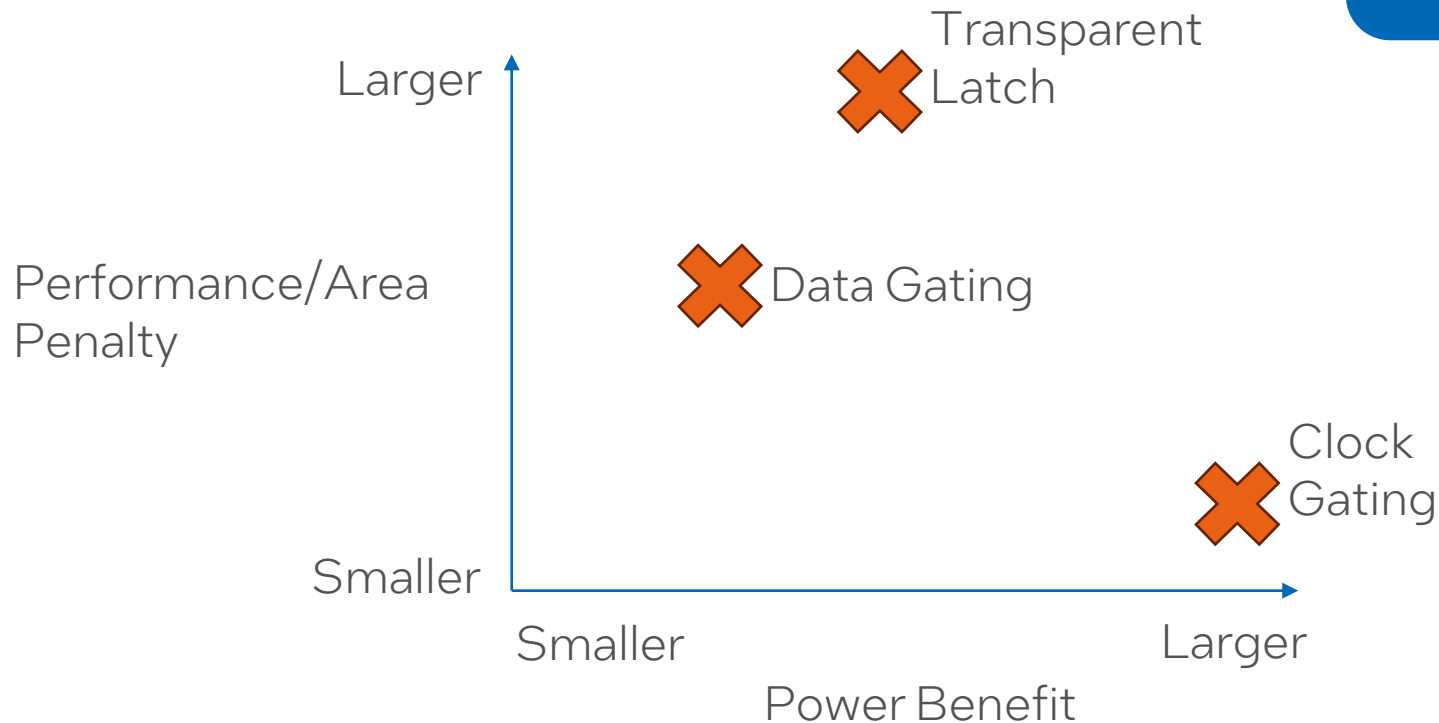


#### Considerations:

- Availability of clock gating signals
- Timing impact & gate count

# Evaluating the Methods

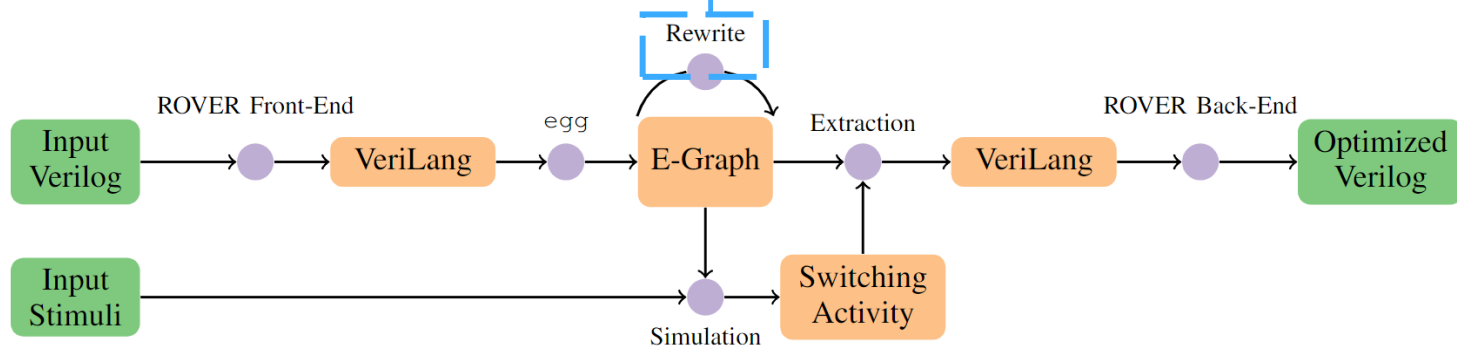
Disclaimer:  
Highly dependent  
upon context!



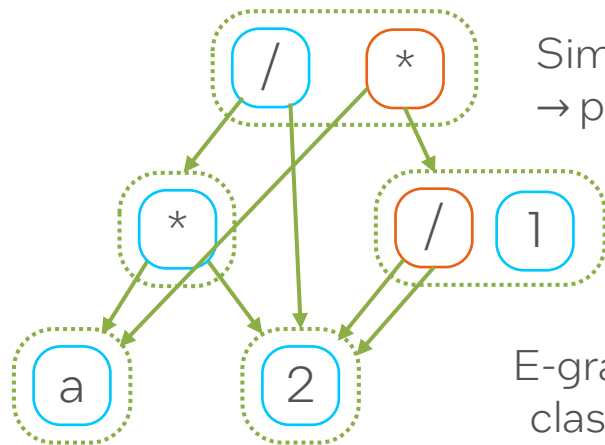
# Combining Power and Arithmetic Optimization

- Arithmetic rewrites – associativity, carry-save
- Logic rewrites – mux tree rearrangement
- Logic arithmetic exchange

All rewrites applied and combined during exploration

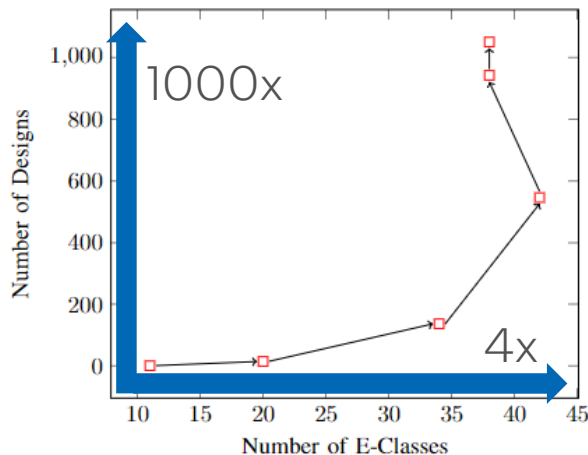


# Computationally Efficient E-Graph Simulation



Simulate one node per e-class  
→ per class switching activity

E-graph simulation scales with  
classes not implementations



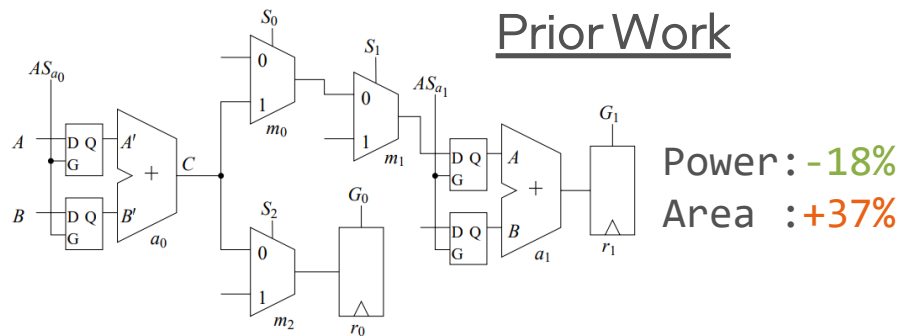
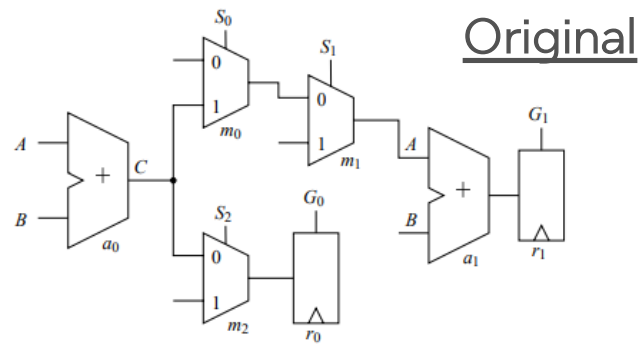
Extraction:

- Nodes in an e-class use more/less energy
- Estimate node cost based on switching activities and operator area
- Extract best using ILP

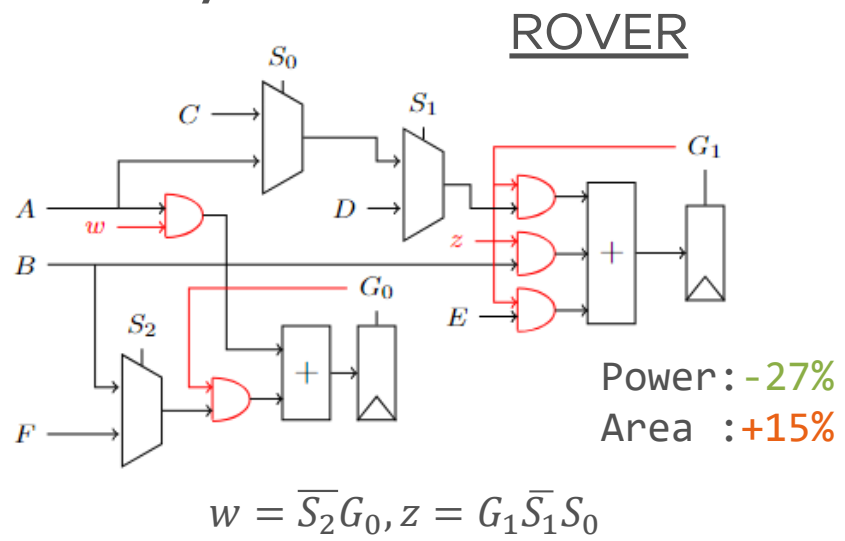
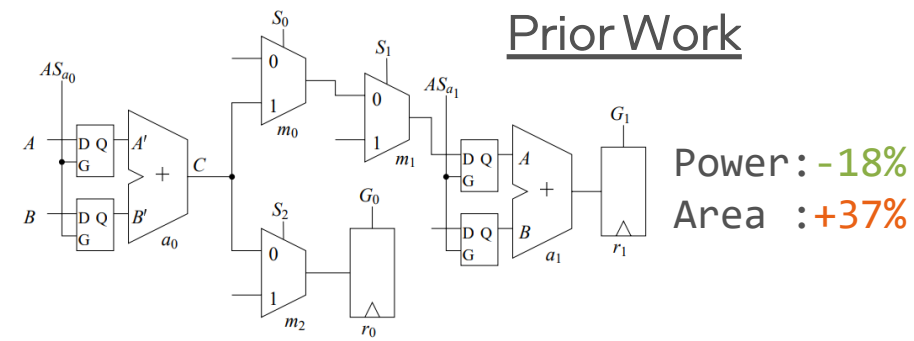
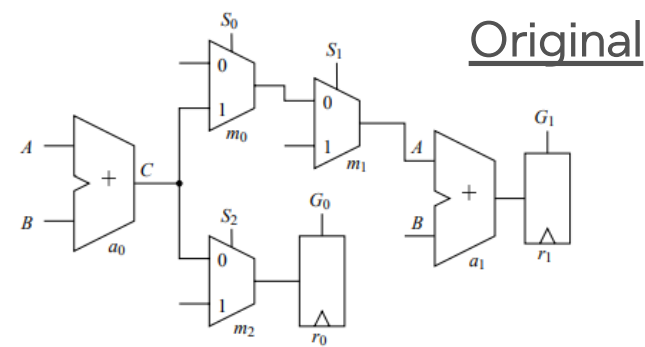
Untapped simulation/  
testing opportunities?



# Datapath Extraction Aware Case Study



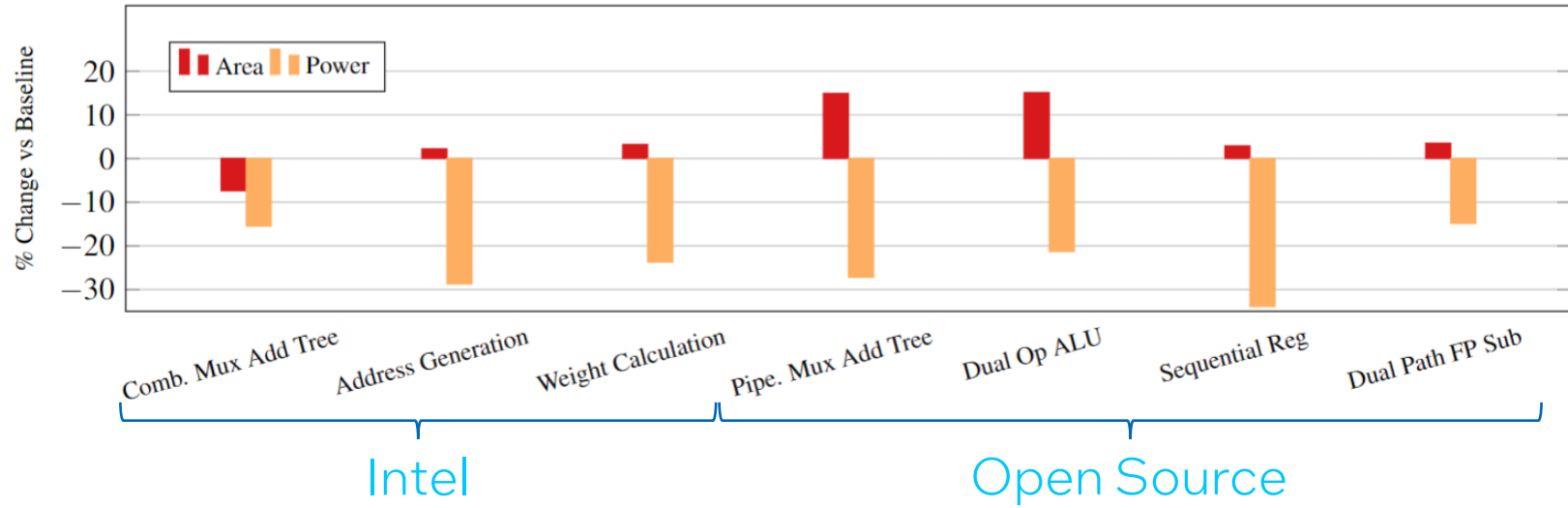
# Datapath Extraction Aware Case Study



Push adders through muxes  
Use 3 input carry-save adder

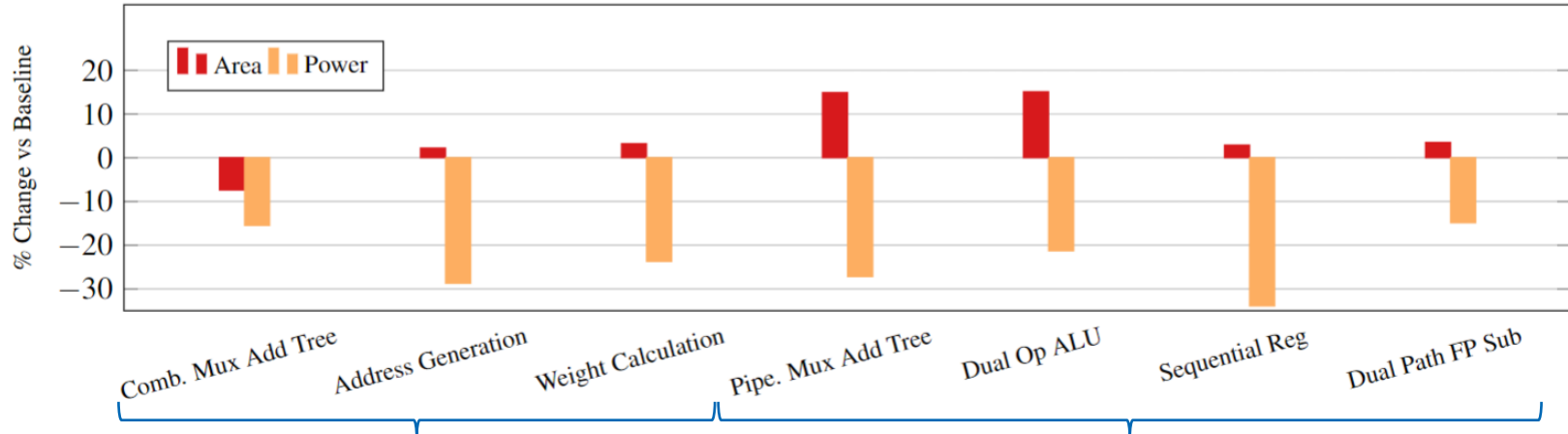
# Results

Up to 34% less power, 24% on average, 5% average area overhead



# Results

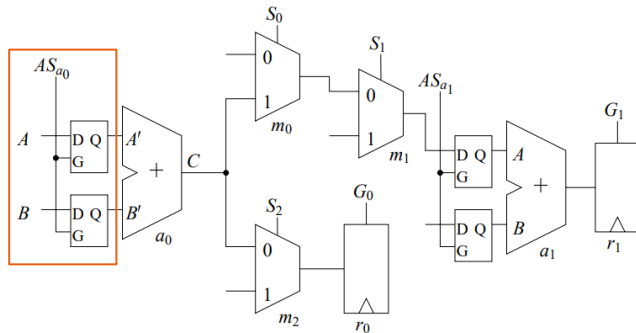
Up to 34% less power, 24% on average, 5% average area overhead



Intel

Open Source

## Future Work



UNREACHABLE!!!

- Resource sharing –not expressible as local equivalence preserving rewrites...

# Backup

## 00

0000

00



# Clock Gate Proof

$$L_i = \text{TREG}(\text{REG}(a_i, en_i), \text{REG}(b_i, en_i)) \text{ and } R_i = \text{REG}(a_i, en_i \& b_i)$$

for all clock cycles  $i$  via induction. First, let

$$p_i = \text{REG}(a_i, en_i) \text{ and } q_i = \text{REG}(b_i, en_i).$$

Suppose  $\forall i \leq k \ L_i = R_i$ , then if  $en_k = 1$ :

$$\begin{aligned} q_{k+1} &= b_k & p_{k+1} &= a_k \\ L_{k+1} &= q_{k+1} ? p_{k+1} : L_k \\ &= b_k ? a_k : L_k \end{aligned}$$

Then, since  $en_k = 1$  and  $R_k = L_k$ ,

$$\begin{aligned} R_{k+1} &= en_k \& b_k ? a_k : R_k \\ &= b_k ? a_k : R_k = L_{k+1} \end{aligned}$$

Now if  $en_k = 0$ , then  $R_{k+1} = R_k = L_k$  and

$$\begin{aligned} q_{k+1} &= q_k & p_{k+1} &= p_k \\ L_{k+1} &= q_k ? p_k : L_k \\ q_k = 1 &\Rightarrow L_k = q_k ? p_k : L_{k-1} = p_k \end{aligned}$$

Therefore  $L_{k+1} = L_k = R_{k+1}$  and hence  $R_{k+1} = L_{k+1}$  for all values of  $en_k$ . Under the zero register initialization assumption it is trivial to prove  $L_0 = R_0$ .